NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS

2019 Year 12 Course Assessment Task 4 (Trial Examination)
Monday August 12, 2019

General instructions

- Working time – 3 hours. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided (on page 11)

SECTION II

- Commence each new question on a new booklet. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: .... **# BOOKLETS USED**: .....

Class (please ✔)

◯ 12MAT.3 – Mrs Gan
◯ 12MAT.4 – Ms Park
◯ 12MAT.5 – Mr Sekaran
◯ 12MAT.6 – Mrs Dupuche

Marker's use only.

QUESTION	1-10	11	12	13	14	15	16	%
MARKS	$\overline{10}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{100}$

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 10 minutes for this section

Mark your answers on the answer grid provided (labelled as page 11).

Questions **Marks**

1. An experiment consists of tossing a coin and then rolling a fair six-sided die. What is the probability of observing a 'head' and a 'six'? **1**

(A) $\frac{1}{2}$ (B) $\frac{1}{35}$ (C) $\frac{7}{12}$ (D) $\frac{1}{12}$

2. What is the solution to the equation $\log_3(2x-5)=2$? **1**

(A) $x=1$ (B) $x=\frac{13}{2}$ (C) $x=\frac{11}{2}$ (D) $x=7$

3. What is the derivative of $\dfrac{\sin x}{\cos x+1}$? **1**

(A) $\dfrac{\sin x+\cos x}{\sin^2 x}$

(B) $\dfrac{\cos^2 x+\cos x-\sin^2 x}{(\cos x+1)^2}$

(C) $\dfrac{1}{\cos x+1}$

(D) $\dfrac{2}{\cos x+1}$

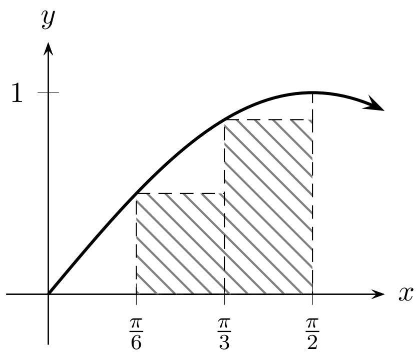
4. What is the result of evaluating $\lim\limits_{x\to 2}\dfrac{x^2+x-6}{x-2}$? **1**

(A) 5 (B) 6 (C) 7 (D) 8

5. What are the coordinates of the turning point to the curve $y=e^x-ex$? **1**

(A) $(0,1)$ (B) $(1,0)$ (C) $(1,e)$ (D) $(e,1)$

6. The area beneath the curve $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{2}$ is approximated by the two rectangles as shown. **1**



What is the approximation to the area?

(A) $\frac{\pi}{2}$ square units

(B) $\frac{2\pi}{3}$ square units

(C) $\frac{\left(1+\sqrt{3}\right)\pi}{12}$ square units

(D) $\frac{\left(1+\sqrt{3}\right)\pi}{6}$ square units

7. The first three terms of an arithmetic series are 5, 9 and 13. **1**

What is the 15th term of the series?

(A) 61 (B) 66 (C) 495 (D) 585

8. What is the solution to $|x-2| < 3$? **1**

(A) $-1 < x < 5$

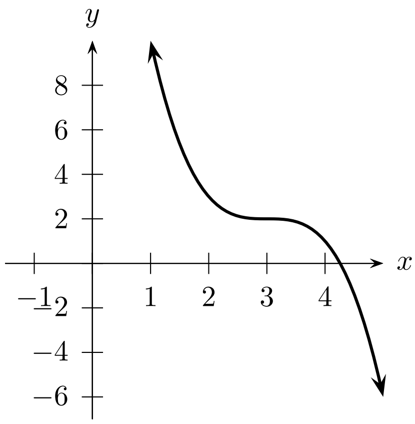
(B) $x < -5$ or $x > 1$

(C) $-5 < x < 1$

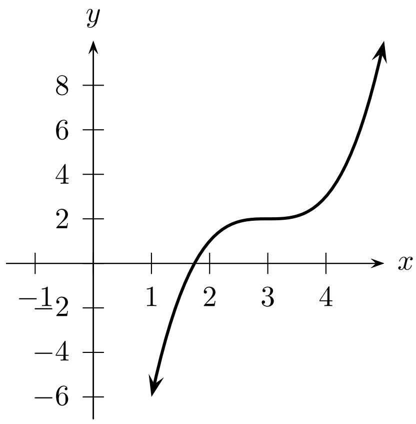
(D) $x < -1$ or $x > 5$

9. Which diagram is the graph of $y = 2 - (x-3)^3$? **1**

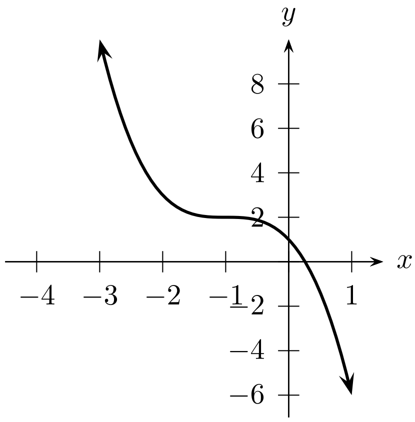
(A)



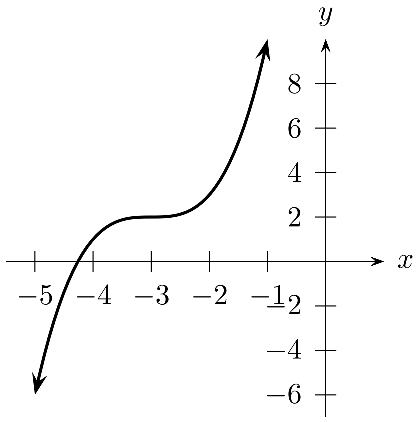
(C)



(B)



(D)



10. For what values of x does the geometric progression $1 + \dfrac{1}{2-x} + \dfrac{1}{(2-x)^2} + \cdots$ has a limiting sum? **1**

(A) $1 < x < 3$

(B) $x < 1$ or $x > 3$

(C) $x \neq 2$

(D) $0 < x < 2$

Examination continues overleaf...

Section II

90 marks
Attempt Questions 11 to 16
Allow approximately 2 hours and 50 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

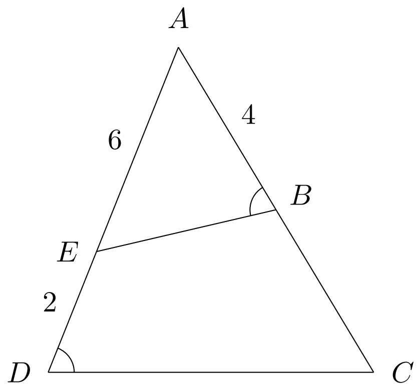
Question 11 (15 Marks) Commence a NEW booklet. **Marks**

(a) Find integers a and b such that $\dfrac{4+\sqrt{3}}{2+\sqrt{3}} \equiv a\sqrt{3}+b$. **2**

(b) Find the domain of the function $y = \dfrac{1}{x^2-4}$. **2**

(c) Find $\displaystyle\int \frac{1}{(x+5)^3}\,dx$. **2**

(d) $\triangle ABE$ is similar to $\triangle ACD$. $AE = 6$, $AB = 4$ and $ED = 2$. **2**



Find the length of BC.

(e) There are 30 students in a class with 24 students exercising by running and 18 exercising at the gym. Each student does at least one type of exercise.

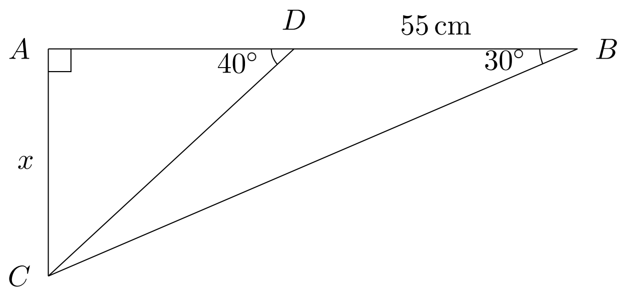
i. One student is chosen at random. What is the probability that the student exercises at the gym? **1**

ii. Two students are chosen at random. What is the probability that both students exercise at the gym? **1**

iii. Two students are chosen at random. What is the probability that both students do both types of exercise? **1**

iv. One student is chosen at random. What is the probability that the student does *not* do both types of exercise? **1**

(f) Find the value of x in the diagram below, where $BD = 55\,\text{cm}$. Give your answer correct to 3 significant figures. **3**



Question 12 (15 Marks) Commence a NEW booklet. **Marks**

(a) The line ℓ_1 passes through the point $(9, -4)$ and has gradient of $\frac{1}{3}$.

i. Find the equation of ℓ_1 in the form $ax + by + c = 0$, where a, b and c are integers. **2**

ii. Line ℓ_2 passes through the origin and has gradient -2. **2**

The lines ℓ_1 and ℓ_2 intersect at the point P. What are the coordinates of P?

iii. Given that ℓ_1 crosses the y axis at the point C. **2**

Calculate the exact area of $\triangle OPC$.

(b) Differentiate with respect to x:

i. $\left(4x^3 - x\right)^7$. **1**

ii. $e^x \cos 4x$. **2**

iii. $\dfrac{x^2 + 2}{3x - 4}$ **2**

(c) Consider the parabola $4y = x^2 - 2x + 5$.

i. Find the coordinates of the vertex. **2**

ii. Find the coordinates of the focus. **1**

iii. Find the values of x such that the function $4y = x^2 - 2x + 5$ is increasing. **1**

Question 13 (15 Marks) Commence a NEW booklet. **Marks**

(a) i. What is the period of the function $y = 4\sin 2x$? **1**

ii. Sketch the function $y = 4\sin 2x + 1$ for $-\pi \le x \le \pi$. **2**

(b) i. Show that $\dfrac{x+2}{5x^2+7x-6} = \dfrac{1}{5x-3}$ **1**

ii. Hence find the value of k such that **3**

$$\int_1^k \frac{x+2}{5x^2+7x-6}\,dx = \frac{1}{5}\ln 6$$

(c) A curve has gradient function with equation

$$\frac{dy}{dx} = 6(x-1)(x-2)$$

i. If the curve passes through the point $(0, -3)$, what is the equation of the curve? **2**

ii. Find the coordinates of the stationary points. **1**

iii. Determine the nature of the stationary points. **1**

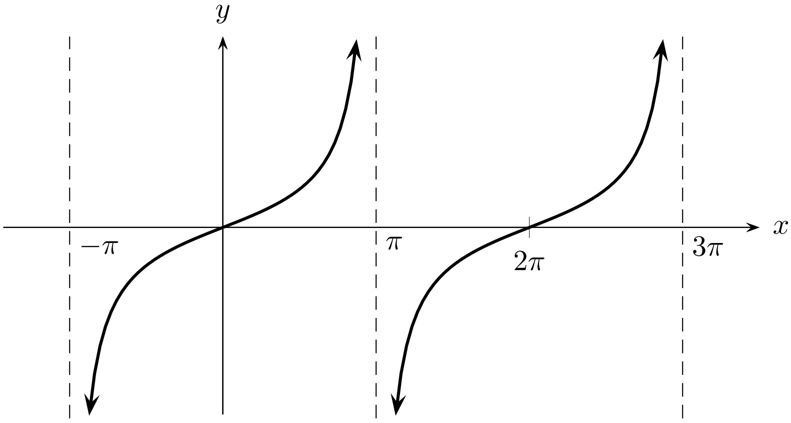
iv. Find the coordinates of the point of inflexion. **2**

v. Sketch the curve, showing the essential features. **2**

Examination continues overleaf...

Question 14 (15 Marks) Commence a NEW booklet. **Marks**

(a) The graph of $f(x) = \tan\dfrac{x}{2}$ from $x = -\pi$ to $x = 3\pi$ is shown below.



i. Find $f'\left(\dfrac{\pi}{2}\right)$. **1**

ii. Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = \dfrac{\pi}{2}$. **2**

iii. Find all the points on the graph of $y = f(x)$ in the given domain, where the gradient equals to 1. **2**

(b) i. Copy and complete the table below for $y = \sqrt{5^x + 2}$ in your writing booklet. **1**

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y			2.646		

ii. Use the Trapezoidal Rule to find an approximation for the value of **2**

$$\int_0^2 \sqrt{5^x + 2}\, dx$$

(c) The second and fifth terms of a geometric series are 750 and -6 respectively.

i. Find the common ratio, and the first term of the series. **2**

ii. What is the limiting sum of the series? **1**

(d) The quadratic equation $3x^2 + 9x + 1 = 0$ has roots α and β. Find the values of:

i. $\alpha + \beta$. **1**

ii. $\alpha\beta$. **1**

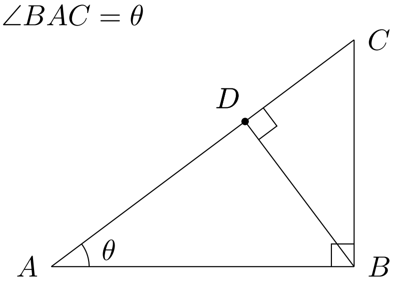
iii. $4\alpha\beta^2 + 4\alpha^2\beta$. **1**

iv. $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$. **1**

Examination continues overleaf...

Question 15 (15 Marks) Commence a NEW booklet. **Marks**

(a) $\triangle ABC$ is right angled at B. D is a point on AC such that BD is perpendicular to AC. Let $\angle BAC = \theta$



NOT TO SCALE

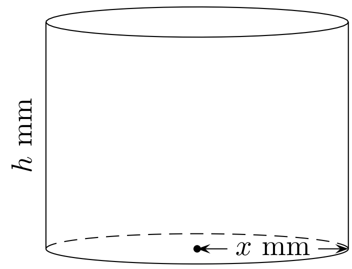
You are given that $6AD + BC = 5AC$.

i. Show that $6\cos\theta + \tan\theta = 5\sec\theta$ **1**

ii. Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$ **1**

iii. Find the value of θ, correct to the nearest degree. **2**

(b) A closed cylinder with base radius x millimetres and height h millimetres is shown. The volume of the cylinder is $60\,\text{mm}^3$.



i. Find an expression for h in terms of x. **1**

ii. Show that the surface area A mm^2 of the cylinder is given by **1**

$$A = \frac{120}{x} + 2\pi x^2$$

iii. Find the value of x which minimises the surface area of the cylinder. **3**

iv. Calculate the minimum surface area of the cylinder, correct to the nearest square millimetres. **1**

(c) A biased coin is tossed three times. The probability of obtaining a head is

$$P(H) = \frac{2}{5}$$

i. Find the probability of obtaining two heads and a tail. **1**

ii. Find the probability of obtaining at least one head. **1**

(d) Find the value of m such that the curves $y = \dfrac{x^4}{m}$ and $y = -3x^3 - 2x^2$ intersect at more than two points. **3**

Examination continues overleaf...

Question 16 (15 Marks) Commence a NEW booklet. **Marks**

(a) Aiden borrows \$15 000 to purchase solar panels for his home. The interest is calculated monthly at the rate of 6% per annum, and is compounded monthly. He intends to repay the loan in monthly instalments of $\$M$.

i. How much does Aiden owe at the end of the first month, before he makes his first repayment? **1**

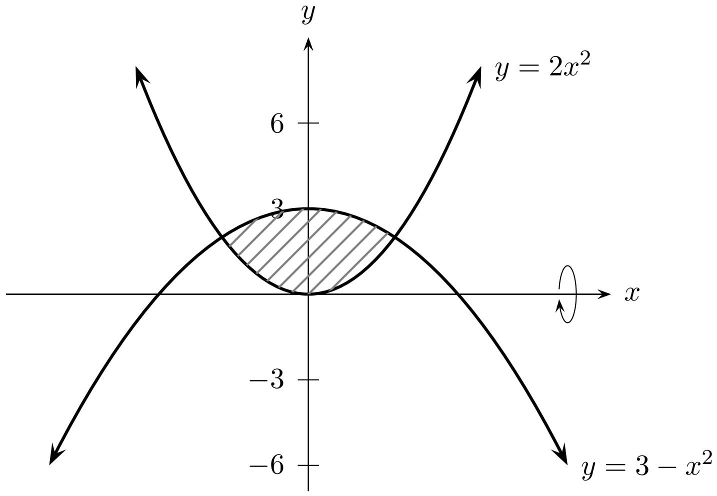
ii. Let A_n be the amount of money owing after n repayments. Show that when $n = 3$, **2**

$$A_3 = \left(15\,000 \times 1.005^3\right) - M\left(1 + 1.005 + 1.005^2\right)$$

iii. After two years of repaying the loan, Aiden still owes \$10 000 on the loan. **3**

What is the amount of the monthly repayment?

(b) The graphs of $y = 2x^2$ and $y = 3 - x^2$ are shown.



i. Find the coordinates of the points of intersection between the two curves. **1**

ii. The shaded region is rotated about the x axis. Find the *exact* volume of the solid formed. **3**

(c) On a factory production line, a tap opens and closes to fill containers with liquid. As the tap opens, the rate of flow R increases for the first 10 seconds according to the rule $R = \dfrac{3t}{25}$, where R is measured in litres per second.

The rate of flow then remains constant until the tap begins to close.

i. Show that while the tap is still fully open, the volume in the container at any time is given by **3**

$$V = \frac{6}{5}(t - 5)$$

ii. For how many seconds must the tap remain fully open in order to exactly fill a 120 L container without any spillage? **2**

End of paper.

2019 HSC Trial Mathematics (2U) Solutions

Section 1

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
D	D	C	A	B	C	A	A	A	B

Question 11

a) $\frac{4+\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{(4+\sqrt{3})(2-\sqrt{3})}{4-3}$

$= 8-2\sqrt{3}-3$

$= 5-2\sqrt{3}$ ✓

$\therefore a=-2$, $b=5$ ✓ (2)

b) $x^2-4 \neq 0$

$x \neq \pm 2$ ✓

$D: \{x \in R, x \neq \pm 2\}$ ✓ (2)

c) $\int (x+5)^{-3}\,dx$

$= \frac{(x+5)^{-2}}{-2} + C$ ✓

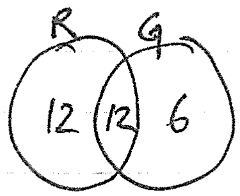
$= -\frac{1}{2(x+5)^2} + C$ ✓ (2)

d) $\frac{BC+4}{6} = \frac{2+6}{4}$ ✓

$BC+4 = 12$

$BC = 8$ ✓ (2)

e)



i) $\frac{18}{30} = \frac{3}{5}$ ✓ (1)

ii) $\frac{18}{30} \times \frac{17}{29} = \frac{51}{145}$ ✓ (1)

Q11 (cont)

e) iii) $\frac{12}{30} \times \frac{11}{29} = \frac{22}{145}$ ✓ (1)

iv) $1 - \frac{12}{30} = \frac{3}{5}$ ✓ (1)

f) $\angle DCB = 40° - 30°$

$= 10°$

In $\triangle BCD$,

$\frac{DC}{\sin 30°} = \frac{55}{\sin 10°}$ ✓

$DC = \frac{55}{2\sin 10°}$

In $\triangle ADC$,

$\sin 40° = \frac{x}{DC}$ ✓

$x = \frac{55 \sin 40°}{2 \sin 10°}$

$= 101.7958\ldots$

$= 102$ cm (3 sig fig.) ✓ (3)

Question 12

a) i) $y-(-4) = \frac{1}{3}(x-9)$ ✓

$3y+12 = x-9$

$l_1: x-3y-21=0$ ✓ (2)

ii) $l_2: y=-2x$

$x-3(-2x)-21=0$ ✓

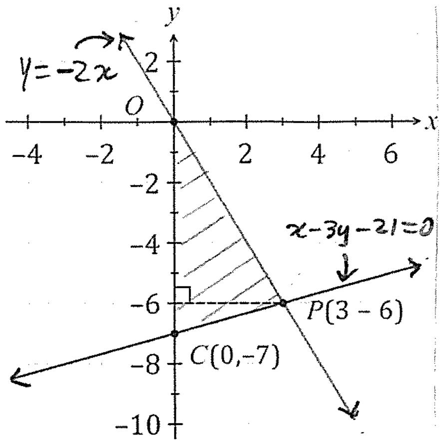
$7x=21$

$x=3$

$\therefore P$ is $(3,-6)$ ✓ (2)

Q12 (cont)

a) (ii)



At C, $-3y - 21 = 0$

$y = -7$

$\therefore C(0,-7)$ ✓

Area of $\triangle OPC$

$= \frac{1}{2} \times 7 \times 3$

$= 10\frac{1}{2}$ sq. units ✓ (2)

b) i) $\frac{d}{dx}(4x^3 - x)^7$

$= 7(4x^3 - x)^6(12x^2 - 1)$ ✓ (1)

ii) $\frac{d}{dx}(e^x \cos 4x)$

$= e^x(-4\sin 4x) + (\cos 4x)e^x$ ✓

$= e^x(\cos 4x - 4\sin 4x)$ ✓ (2)

iii) $\frac{d}{dx}\left(\frac{x^2+2}{3x-4}\right)$

$= \frac{(3x-4)(2x) - (x^2+2)(3)}{(3x-4)^2}$ ✓

$= \frac{6x^2 - 8x - 3x^2 - 6}{(3x-4)^2}$

$= \frac{3x^2 - 8x - 6}{(3x-4)^2}$ ✓ (2)

c) i) $4y = x^2 - 2x + 5$ (2)

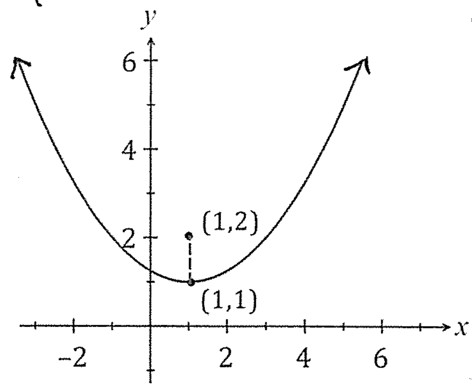
$x^2 - 2x = 4y - 5$

$(x-1)^2 = 4(y-1)$ ✓

$\therefore$ vertex is $(1,1)$ ✓

ii) focus is $(1,2)$ ✓ (1)

iii)

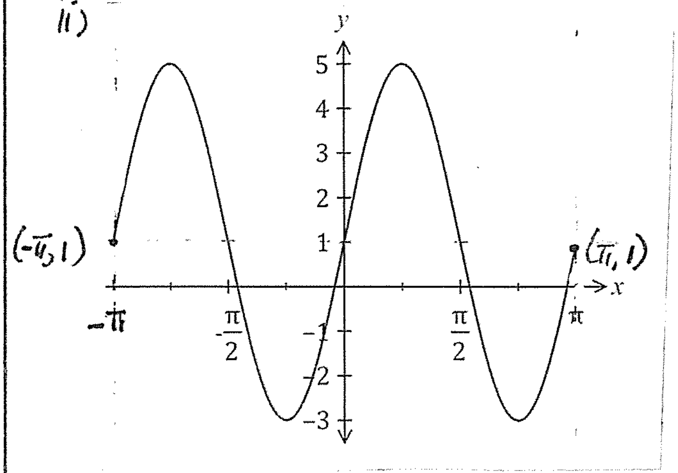


$x \geq 1$ ✓ (1)

Question 13

a) i) π (1)

ii)



shape ✓

y intercept, endpoints } labelled ✓ (2)

Q13 (cont)

b) i) $\frac{x+2}{5x^2+7x-6} = \frac{x+2}{(5x-3)(x+2)}$ ✓

$= \frac{1}{5x-3}$ (1)

ii) $\int_1^k \frac{x+2}{5x^2+7x-6}\,dx$

$= \frac{1}{5}\int_1^k \frac{5}{5x-3}\,dx$

$= \frac{1}{5}\left[\ln(5x-3)\right]_1^k$ ✓

$= \frac{1}{5}\left[\ln(5k-3) - \ln 2\right]$

$= \frac{1}{5}\ln\frac{5k-3}{2}$

$\frac{5k-3}{2} = 6$ ✓

$5k-3 = 12$

$k = 3$ ✓ (3)

c) i) $\frac{dy}{dx} = 6(x-1)(x-2)$
$= 6x^2 - 18x + 12$

$y = \int(6x^2-18x+12)\,dx$
$= 2x^3 - 9x^2 + 12x + C$ ✓

Sub $(0,-3)$,
$-3 = C$

$\therefore y = 2x^3 - 9x^2 + 12x - 3$ ✓ (2)

ii) $(1,2)$, $(2,1)$ ✓ (1)

iii) $\frac{d^2y}{dx^2} = 12x - 18$

At $(1,2)$, $\frac{d^2y}{dx^2} < 0$

At $(2,1)$, $\frac{d^2y}{dx^2} > 0$ (1)

$\therefore$ (1,2) is a maximum turning point ✓
and (2,1) is a minimum turning point.

iv) $12x - 18 = 0$

$x = 1\frac{1}{2}$

$y = 1\frac{1}{2}$

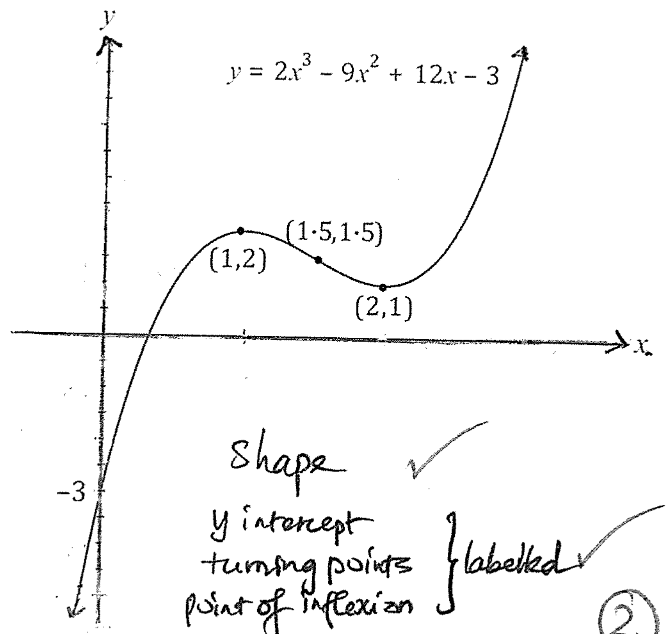
x	1.4	1.5	1.6
$\frac{d^2y}{dx^2}$	-1.2	0	1.2

✓

$\frac{d^2y}{dx^2}$ changes sign around $x = 1.5$

$\therefore (1\frac{1}{2}, 1\frac{1}{2})$ is a point of inflexion. ✓ (2)

v)



Shape ✓

y intercept
turning points } labelled ✓
point of inflexion

(2)

Question 14

a) i) $f(x) = \tan\frac{x}{2}$

$f'(x) = \frac{1}{2}\sec^2(\frac{x}{2})$

$f'(\frac{\pi}{2}) = \frac{1}{2}\sec^2(\frac{\pi}{4})$

$= 1$ ✓ ①

ii) $x = \frac{\pi}{2}$, $y = 1$

∴ equation of normal is:

$y - 1 = -1(x - \frac{\pi}{2})$ ✓

$y = -x + \frac{\pi}{2} + 1$ ✓ ②

iii) $f'(x) = 1$

$\frac{1}{2}\sec^2(\frac{x}{2}) = 1$

$\sec^2(\frac{x}{2}) = 2$

$\cos\frac{x}{2} = \pm\frac{1}{\sqrt{2}}$ ✓

∴ the points are:

$(-\frac{\pi}{2}, -1)$, $(\frac{\pi}{2}, 1)$, $(\frac{3\pi}{2}, -1)$, $(\frac{5\pi}{2}, 1)$ ✓ ②

b) i)

x	0	0.5	1	1.5	2
y	1.732	2.058	2.646	3.630	5.196

①

ii) $\frac{0.5}{2}[1.732 + 5.196 + 2(2.058 + 2.646 + 3.630)]$ ✓

$= 5.899$ ✓ ②

c) $ar = 750$ — ①

$ar^4 = -6$ — ②

$\frac{②}{①}$, $r^3 = -\frac{1}{125}$

$r = -\frac{1}{5}$ ✓

Sub into ①,

$a = -3750$ ✓ ②

ii) $\frac{-3750}{1-(-\frac{1}{5})}$

$= -3125$ ✓ ①

d) i) $-\frac{9}{3} = -3$ ✓ ①

ii) $\frac{1}{3}$ ✓ ①

iii) $4\alpha\beta^2 + 4\alpha^2\beta$

$= 4\alpha\beta(\beta + \alpha)$

$= 4 \times \frac{1}{3}(-3)$

$= -4$ ✓ ①

iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{(-3)^2 - 2(\frac{1}{3})}{(\frac{1}{3})^2}$

$= 75$ ✓ ①

Question 15

a) i) In $\triangle ABD$, $\cos\theta = \frac{AD}{AB}$

∴ $AD = AB\cos\theta$

In $\triangle ABC$, $\tan\theta = \frac{BC}{AB}$

∴ $BC = AB\tan\theta$

$\sec\theta = \frac{AC}{AB}$

∴ $AC = AB\sec\theta$ ✓

$6AD + BC = 5AC$

∴ $6AB\cos\theta + AB\tan\theta = 5AB\sec\theta$

$6\cos\theta + \tan\theta = 5\sec\theta$ ①

ii) $6\cos\theta + \frac{\sin\theta}{\cos\theta} = \frac{5}{\cos\theta}$

$6\cos^2\theta + \sin\theta = 5$ ✓

$6(1 - \sin^2\theta) + \sin\theta = 5$

$6\sin^2\theta - \sin\theta - 1 = 0$ ①

iii) $\sin\theta = \frac{1 \pm \sqrt{1 + 4(6)}}{2(6)}$ ✓

$= \frac{1 \pm 5}{12}$

$= \frac{1}{2}$ or $-\frac{1}{3}$

$\theta = 30°$ ✓ as $0 \leq \theta \leq 90°$ ②

Q 15 (cont)

b) i) $60 = \pi x^2 h$

$h = \dfrac{60}{\pi x^2}$ ✓ (1)

ii) $A = 2\pi r h + 2\pi r^2$

$= 2\pi x \times \dfrac{60}{\pi x^2} + 2\pi x^2$

$= \dfrac{120}{x} + 2\pi x^2$ ✓ (1)

iii) $\dfrac{dA}{dx} = -120x^{-2} + 4\pi x$ ✓

$= 0$

when $4\pi x = \dfrac{120}{x^2}$

$x = \sqrt[3]{\dfrac{30}{\pi}}$ ✓

$\dfrac{d^2A}{dx^2} = 240x^{-3} + 4\pi$

$= 240 \times \dfrac{\pi}{30} + 4\pi$ ✓

$= 12\pi > 0$

$\therefore$ A is minimum when

$x = \sqrt[3]{\dfrac{30}{\pi}}$ (3)

iv) Min $A = 120 \times \sqrt[3]{\dfrac{\pi}{30}} + 2\pi\left(\dfrac{30}{\pi}\right)^{\frac{2}{3}}$

$= 84.8428\ldots$

$\doteqdot 85\text{ m}^2$ ✓ (1)

c) i) $P(HHT) + P(HTH) + P(THH)$

$= \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} \times 3$

$= \dfrac{36}{125}$ ✓ (1)

ii) $P(1H)$

$= 1 - (TTT)$

$= 1 - \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \dfrac{98}{125}$ ✓ (1)

d) $\dfrac{x^4}{m} = -3x^3 - 2x^2$

$x^4 + 3mx^3 + 2mx^2 = 0$

$x^2(x^2 + 3mx + 2m) = 0$

$x = 0$ is a solution ✓

For two more solutions,

$x^2 + 3mx + 2m = 0$

$\Delta > 0$

$9m^2 - 4(1)(2m) > 0$ ✓

$m(9m - 8) > 0$

$m < 0$ or $m > \frac{8}{9}$ ✓ (3)

Question 16

a) i) 15000×1.005

$= \$15075$ ✓ (1)

ii) $A_1 = 15000 \times 1.005 - M$

$A_2 = (15000 \times 1.005 - M)1.005 - M$

$= 15000 \times 1.005^2 - M(1 + 1.005)$ ✓

$A_3 = [15000 \times 1.005^2 - M(1 + 1.005)] \times 1.005 - M$ ✓

$= 15000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)$ (2)

iii) $A_{24} = 15000 \times 1.005^{24} - M(1 + 1.005 + \cdots + 1.005^{23})$ ✓

$10000 = 15000 \times 1.005^{24} - M(1 + 1.005 + \cdots + 1.005^{23})$

$= 15000 \times 1.005^{24} - M \times \dfrac{1(1.005^{24} - 1)}{1.005 - 1}$ ✓

$M = \dfrac{(15000 \times 1.005^{24} - 10000)0.005}{1.005^{24} - 1}$

$= \$271.60$ ✓ (3)

Q16 cont

b) i) $2x^2 = 3 - x^2$

$3x^2 = 3$

$x = \pm 1$

∴ points are intersection are

$(-1, 2)$, $(1, 2)$ ✓ (1)

ii) $V = 2\pi \int_0^1 \left[(3-x^2)^2 - (2x^2)^2\right] dx$ ✓

$= 2\pi \int_0^1 (9 - 6x^2 + x^4 - 4x^4) dx$

$= 2\pi \int_0^1 (-3x^4 - 6x^2 + 9) dx$

$= 2\pi \left[-\frac{3x^5}{5} - 2x^3 + 9x \right]_0^1$ ✓

$= 2\pi \left(-\frac{3}{5} - 2 + 9 \right)$

$= \frac{64\pi}{5}$ cu. units ✓ (3)

Question 16 c i

When $0 < t \leq 10$, $\quad \frac{dV}{dt} = \frac{3}{25}t$

So, $\quad V = \frac{1}{2} \times \frac{3}{25}t^2 + c_1$

And $\quad t = 0, V = 0 \Rightarrow c_1 = 0$

$\therefore V = \frac{3}{50}t^2$ ✓

When $t = 10$, $\quad V = \frac{3}{50}(10)^2 \quad = 6$ L ✓

And when $t \geq 10$ $\quad \frac{dV}{dt} = \frac{3}{25} \times 10 \quad = \frac{6}{5}$ L/s

(actually $10 \leq t < t_1$ if you consider part ii)

So when $t \geq 10$ $\quad V = \frac{6}{5}t + c_2$

(actually $10 \leq t < t_1$ if you consider part ii)

And $\quad t = 10, V = 6 \Rightarrow c_2 = -6$

$\therefore V = \frac{6}{5}t - 6 \quad = \frac{6}{5}(t-5)$

as required. ✓

Alternatively (1):

$V \quad = \int_0^{10} \frac{3t}{25} dt$

$= \frac{3}{25} \int_0^{10} t \, dt$

$= 6$ L

When $t > 10$, $\quad \frac{dV}{dt} = \frac{3}{25} \times 10 \quad = \frac{6}{5}$ L/s

(actually $10 \leq t < t_1$ if you consider part ii)

So when $t > 10$, $\quad V = V_{up\ to\ 10s} + V_{after\ 10s}$

(actually $10 \leq t < t_1$ if you consider part ii)

$= 6 + \frac{dV}{dt} \times t$

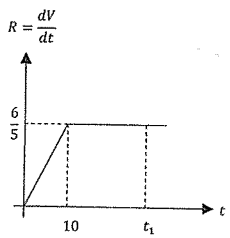
$= 6 + \frac{6}{5} \times (t - 10)$

$= 6 + \frac{6}{5}t - 12$

$= \frac{6}{5}t - 6$

$= \frac{6}{5}(t-5)$

Alternatively (2):



$V = \int_0^t R \, dt$

$= Area\ under\ R(t)$

$= \frac{6}{5}t - \frac{1}{2}\left(\frac{6}{5} \times 10\right)$

$= \frac{6}{5}t - 6$

$= \frac{6}{5}(t-5)$

Alternatively (3):

$V \quad = \frac{6}{5}t - \int_0^{10} R \, dt$

$= \frac{6}{5}t - \int_0^{10} \frac{3}{25}t \, dt$

$= \frac{6}{5}t - 6$

$= \frac{6}{5}(t-5)$

Question 16 c ii

Two ways you can fill the container:

1. Place container under closed tap → open tap → **close the tap to fill to 120 L**

t		0 s	10 s	t_1 s	t_2 s
Tap is	Closed	Opening	Fully open	Closing	Closed
$\frac{dv}{dt} = R$	0	$\frac{3t}{25}$	$\frac{6}{5}$	$-\frac{3t}{25}$	0
V		0 L	6 L	120 − 6 = 114 L	120 L

Solving for when $V = 114$ L, $\quad 114 = \frac{6}{5}(t-5)$

$95 = t - 5$

$t = 100$ ✓

∴ Tap has been fully open for $100 - 10 = 90$ s ✓

2. Place container under closed tap → open tap → **remove container from under the tap when it reaches 120 L**

t		0 s	10 s	t_1 s	
Tap is	Closed	Opening	Fully open		
$\frac{dv}{dt} = R$	0	$\frac{3t}{25}$	$\frac{6}{5}$		
V		0 L	6 L	120 L	

Solving for when $V = 120$ L, $\quad 120 = \frac{6}{5}(t-5)$

$100 = t - 5$

$t = 105$ ✓

∴ Tap has been fully open for $105 - 10 = 95$ s ✓